Stated analytically these conditions are

$$\Gamma_p = \frac{-jB_p}{2+jB_p},$$

$$ke^{+2j\phi} = \frac{1-G-jB_k}{1+G+jB_k},$$

$$\Gamma_2' e^{-2j\theta} = \frac{1-G-jB'}{1+G+jB'},$$

$$B_p = B_k - B' \qquad [B_p \text{ positive}].$$

Substitution of these conditions in the expression for S_{11} above does give the result $S_{11}=k$. Since the S_{11} coefficient can be made equal to any arbitrary value k when the fixed port-1 reflection is absent, it is obvious that the S_{11} coefficient for the complete system can be made equal to any arbitrary value a. The value is

$$a = \frac{\Gamma_1''(1-k\Gamma_1') + T_1^2 k}{1-k\Gamma_1'},$$

or

$$k = \frac{\Gamma_1'' - a}{\Gamma_1''\Gamma_1' - T_1^2 - \Gamma_1' a}.$$

The slide-screw tuner can now be represented as a two-port device with three coefficients as shown in Fig. 6(e), and we can conclude that Γ_1 or Γ_2 can be made any arbitrary value by adjustment of the probe.

ACKNOWLEDGMENT

The author wishes to thank Dr. P. D. Lacy for his many helpful suggestions.

Stepped Transformers for Partially Filled Transmission Lines*

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Summary—**In recent years, partially-filled transmission lines have been used to improve the characteristics of various ferrite and garnet devices. This paper presents a generalized outline for determining the approximate effective guide wavelength and characteristic impedance of two types of (dielectric-loaded) partially-filled transmission line. The results are used to determine the geometries required for the design of optimum stepped transmission line transformers. The stepped transitions are designed to yield a Tchebycheff-type response for any given bandwidth. The measured results for stepped transitions in partially filled coaxial line and partially filled double-ridge waveguide are presented. The data are found to approximate the theory closely.**

I. INTRODUCTION

DIELECTRIC-loading techniques[1,2] are frequently used to improve the characteristics of certain ferrite and garnet devices. It has been shown that one method of obtaining a nonreciprocal device in conventional coaxial or strip transmission line is to distort the dominant (TEM or Quasi-TEM) mode by use of a dielectric material.[3,4] The nonreciprocal characteristics of double-ridge waveguide components are also often improved by supplementing the ferrite or garnet with a dielectric material.[5] The addition of this dielectric material changes the characteristic impedance of the transmission line, and this in turn introduces the problem of matching. Cohn has shown that for a given number of steps a Tchebycheff stepped transformer design will give the minimum possible VSWR for a specified bandwidth.[6] Three stepped transitions, in partially filled transmission line, are shown in Fig. 1. The use of a stepped transition will normally 1) substantially reduce the inherent VSWR of a device, 2) enable a specific unit to be made considerably shorter or 3) result in a compromise between the two.

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[1] P. H. Vartanian, J. L. Melchor, and W. P. Ayres, "Broadband-ing ferrite microwave isolators," 1956 IRE NATIONAL CONVENTION RECORD, pt. 5, pp. 79–83.

[2] E. A. Ohm, "A broadband microwave circulator," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-4, pp. 210–217; October, 1956.

[3] B. J. Duncan, L. Swern, K. Tomiyasu, and J. Hannwacker, "Design considerations for broadband ferrite coaxial line isolators," PROC. IRE, vol. 45, pp. 483–490; April, 1957.

[4] D. Fleri and G. Hanley, "Nonreciprocity of dielectric loaded TEM mode transmission lines," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-7, pp. 23–27; January, 1959.

[5] E. Grimes, D. Bartholomew, D. Scott, and S. Sloan, "Broad-band ridge waveguide ferrite devices," presented at the IRE National Symposium on Microwave Theory and Techniques, Harvard University, Cambridge, Mass.; June 1–3, 1959.

[6] S. B. Cohn, "Optimum design of stepped transmission line transformers," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-3, pp. 16–21; April, 1955.

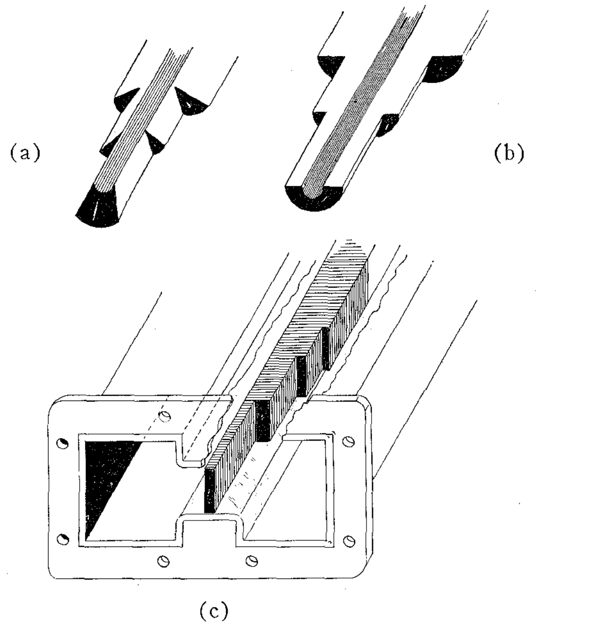


Fig. 1—Step transitions in partially filled transmission line. (a) Coaxial step transition for changing angle, (b) coaxial step transition for changing diameter, (c) double-ridge waveguide step transition.

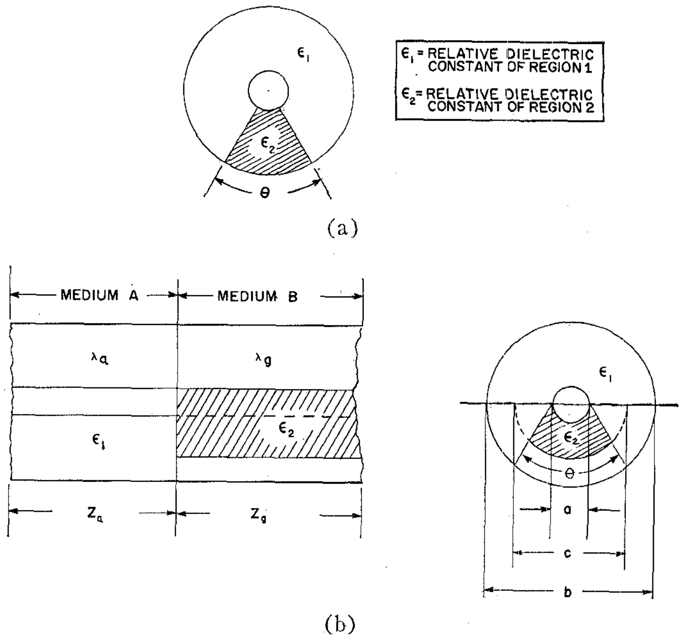


Fig. 2—Cross section of partially filled coaxial transmission lines.

It has been shown that in a partially filled coaxial line, as shown in Fig. 2(a), the guide wavelength may be obtained as a function of the dielectric wedge angle θ.[7] Normally, it is difficult to machine a dielectric wedge, especially for small angles and small transmission lines. Fig. 2(b) shows another type of partially filled coaxial transmission line.

This paper describes a method for computing the approximate effective guide wavelength and characteristic impedance for the types of partially filled transmission line shown in Fig. 1. It also outlines a generalized procedure for designing step transitions in each type of partially filled transmission line.

II. Theoretical Design Considerations

A. Partially Filled Coaxial Transmission Line

1) *Guide Wavelength and Characteristic Impedance:* Fig. 2(b) shows a partially filled coaxial transmission line. Neglecting fringing effects, and under static conditions, the capacitance per unit length for the two cross-sectional areas of medium B may be written as[8]

$$C_1 = \frac{360\epsilon_1}{\ln\frac{b}{a}} - \frac{\theta\epsilon_1}{\ln\frac{b}{a}} \quad \text{and} \quad C_2 = \frac{\theta\epsilon_1\epsilon_2}{\epsilon_1 \ln\frac{c}{a} + \epsilon_2 \ln\frac{b}{c}}.$$

[7] D. J. Angelakos, "A coaxial line filled with two non-concentric dielectrics," IRE Trans. on Microwave Theory and Techniques, vol. MTT-2, pp. 39–44; July, 1954.

[8] J. D. Kraus, "Electromagnetics," McGraw-Hill Book Co., Inc., New York, N. Y., pp. 74–75; 1953.

The total equivalent capacitance is

$$C_e = C_1 + C_2$$

$$= C\left[\frac{\theta}{360}\left(\frac{\epsilon_2 \ln\frac{b}{a}}{\epsilon_1 \ln\frac{c}{a} + \epsilon_2 \ln\frac{b}{c}} - 1\right) + 1\right] \quad (1)$$

where

$$C = \frac{360\epsilon_1}{\ln\frac{b}{a}}.$$

In medium A, the equivalent phase velocity, V, and propagating wavelength, λ_a, are

$$V = f\lambda_a = \frac{1}{\sqrt{LC}}.$$

In medium B, the equivalent phase velocity, V_e, and propagating wavelength, λ_g, are

$$V_e = f\lambda_g = \frac{1}{\sqrt{LC_e}}.$$

Thus,

$$V_e = f\lambda_g = \frac{f\lambda_a}{\sqrt{\frac{\theta}{360}\left(\frac{\epsilon_2 \ln\frac{b}{a}}{\epsilon_1 \ln\frac{c}{a} + \epsilon_2 \ln\frac{b}{c}} - 1\right) + 1}}.$$

For coaxial line,

$$\frac{\lambda_g}{\lambda_a} = \frac{Z_g}{Z_a}. \tag{2}$$

Therefore,

$$\frac{\lambda_g}{\lambda_a} = \frac{Z_g}{Z_a} = \frac{1}{\sqrt{\frac{\theta}{360}\left[\frac{\epsilon_2 \ln \frac{b}{a}}{\epsilon_1 \ln \frac{c}{a} + \epsilon_2 \ln \frac{b}{c}} - 1\right] + 1}} \tag{3}$$

with the parameters as described in Fig. 2(b). In this equation, λ_g is the effective guide wavelength and Z_g the characteristic impedance of the partially filled coaxial transmission line.

Fig. 3 shows the variation of Z_g/Z_a and λ_g/λ_a as a function of b/c, with $Z_a = 50$ ohms, $\theta = 180°$ and $\epsilon_1 = 1.00$, for various values of the relative dielectric constant ϵ_2.

2) *Step Transition Design Procedure:*

a) *Intermediate dielectric radii:* Fig. 4 shows a partially filled coaxial transmission line step transition. The ratio of the two terminating impedances may be defined as

$$C^S = \frac{Z_{n+1}}{Z_1} \tag{4}$$

where

C = a dimensionless constant,
S = the sum of a certain set of a_m constants as outlined by Cohn.[6] [See Appendix, Sections A and B-1), below],
Z_{n+1} = the highest impedance,
Z_1 = the lowest impedance.
Z_n = the nth impedance.

When the value of C has been determined, the intermediate impedance values are then computed as:

$$\begin{aligned} Z_2 &= Z_1 C^{a_1} \\ Z_3 &= Z_1 C^{a_1+a_2} \\ &\vdots \\ Z_n &= Z_1 C^{a_1+a_2+\cdots+a_{n-1}} \end{aligned} \tag{5}$$

where

$$\frac{Z_n}{Z_{n+1}} = \frac{\lambda_n}{\lambda_{n+1}}.$$

After the intermediate impedance values have been computed, the values of b/c may be obtained from Fig. 3, in which Z_g/Z_a will correspond to the appropriate individual Z_n/Z_{n+1} ratios. Once each b/c ratio is ascertained, the intermediate c values are known.

b) *Transformer length:* Each transformer length for the partially filled coaxial step transition is given by the following formula:

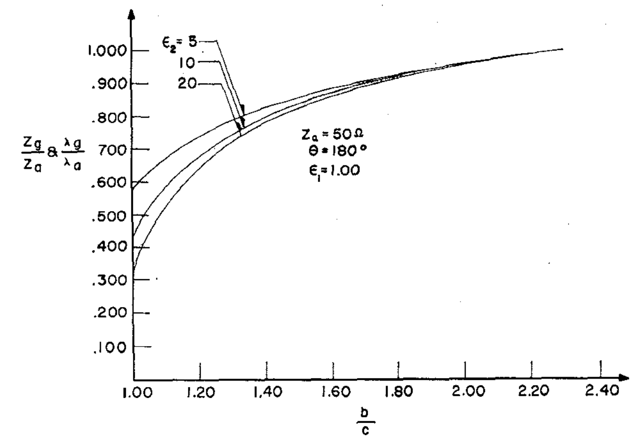


Fig. 3—Normalized guide wavelength and characteristic impedance for partially filled coaxial line.

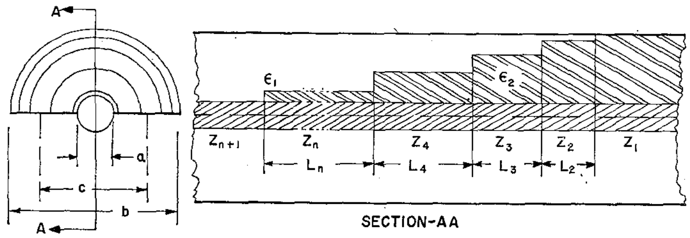


Fig. 4—Partially filled coaxial line step transition.

$$L_n = \frac{\lambda_h \lambda_l}{2(\lambda_h + \lambda_l)} \left(\frac{\lambda_n}{\lambda_{n+1}} \right) \tag{6}$$

where

λ_h = shortest free space wavelength,
λ_l = longest free space wavelength, and

$$\frac{\lambda_n}{\lambda_{n+1}} = \frac{Z_n}{Z_{n+1}}$$

c) *Theoretical VSWR:* The theoretical VSWR[9] for the partially filled coaxial line stepped transformer is given as

$$\text{VSWR} = 1 + \left[\ln \frac{Z_{n+1}}{Z_1} \right] \left[\frac{\left| T_{n-1}\left(\frac{\cos \phi}{\cos \phi_1} \right) \right|}{T_{n-1}\left(\frac{1}{\cos \phi_1} \right)} \right] \tag{7}$$

where

$T_m(x)$ = the Tchebycheff polynomial of the mth degree,
$\phi_1 = 180/1+p$ = electrical spacing of the steps at the low-frequency edge of the band,
ϕ = electrical spacing of the steps,
p is defined in Section A of the Appendix,

[9] S. B. Cohn, private communication to D. Sullivan; January 23, 1959.

B. Partially Filled Double Ridge Waveguide

1) *Guide Wavelength and Characteristics Impedance:* Fig. 5(a) shows a cross-sectional view of a double-ridge waveguide in which the effective dielectric constant ϵ_e completely fills the region between the ridges. Neglecting the discontinuity capacitance C_d, the cutoff frequency has been given by[10]

$$f_c = \frac{1}{2\pi\sqrt{(C_B)\left(\frac{L_A}{2}\right)}} \tag{8}$$

where

f_c = cutoff frequency,
C_B = total capacitance between the ridges,
L_A = total inductance of the side loops.

For Fig. 5(a), (8) may be written as

$$f_c = \frac{1}{2\pi\sqrt{\left(\frac{2d\epsilon_e}{g}\right)\left(\frac{\mu l h}{2}\right)}} \tag{9}$$

where

ϵ_e = effective dielectric constant between the ridges,
μ = permeability of the loop region,

and the other parameters are as shown in Fig. 5(a).

Fig. 5(b) shows a double-ridge waveguide partially filled with a dielectric of thickness t. Considering capacitance C_d, but neglecting the capacitance at the dielectric edges, we may write the total capacitance for Fig. 5 as

$$C_T = C_d + C_1 + C_2 = C_d + \frac{(2d - t)\epsilon_1}{g} + \frac{t\epsilon_2}{g}$$

$$= C_d + \frac{2d\epsilon_1 + t(\epsilon_2 - \epsilon_1)}{g}. \tag{10}$$

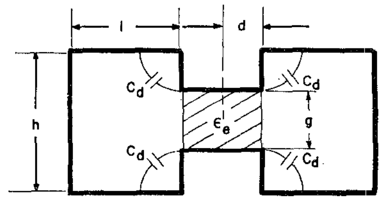
When capacitance C_d is taken into account, substitution of (10) for C_B in (8) yields the cutoff frequency for dielectric thickness t as

$$f_c = \frac{1}{2\pi\sqrt{\left[C_d + \frac{2d\epsilon_1 + t(\epsilon_2 - \epsilon_1)}{g}\right]\left[\frac{\mu l h}{2}\right]}}. \tag{11}$$

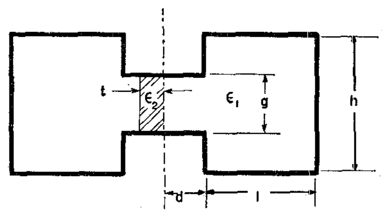
An effective dielectric constant ϵ_e can also be computed for Fig. 5(b) using the formula

$$\epsilon_e = \frac{g}{2d} C_d + \epsilon_1 + \frac{t}{2d}(\epsilon_2 - \epsilon_1). \tag{12}$$

[10] S. Ramo and J. R. Whinnery, "Fields and Waves in Modern Radio," John Wiley and Sons, Inc., New York, N. Y., 2nd ed., pp. 409–410; 1953.



(a)



(b)

Fig. 5—Cross section of a partially filled double-ridge waveguide. *Note:* For DR-37 waveguide, $d = 0.230$ inch, $g = 0.370$ inch, $h = 1.120$ inches, $l = 0.915$ inch, $C_d = 0.222$ $\mu\mu f$/inch.

It is known that the guide wavelength is given by the relationship

$$\lambda_g = \frac{\lambda}{\sqrt{\frac{\epsilon_e}{\epsilon_0} - \left(\frac{f_c}{f}\right)^2}}. \tag{13}$$

Substitution into the above equation for f_c and ϵ_e from (11) and (12) gives the effective guide wavelength for the partially filled double-ridge waveguide as a function of dielectric thickness. Fig. 6 shows the variation of λ_g as a function of frequency for DR-37 waveguide for various dielectric thicknesses. For Fig. 6, C_d was found to be 0.222 $\mu\mu$f/inch; $\epsilon = 1.00$, and $\epsilon_2 = 9.60$.

The characteristic impedance for the geometry shown in Fig. 5(b) may be computed through the use of (11) and (13), using the following formula:[11]

$$Z_0 = Z_{0\infty}\left(\frac{\lambda_g}{\lambda}\right) \tag{14}$$

where

Z_0 = the effective characteristic impedance,
λ_g = the effective guide wavelength,
$Z_{0\infty}$ = the characteristic impedance at infinite frequency for the TE_{10} mode.

The impedance $Z_{0\infty}$ may be readily computed as outlined by Cohn.[11]

2) *Step Transition Design Procedure:*

a) *Intermediate dielectric thicknesses:* Fig. 7 shows a dielectric step transition in double-ridge waveguide. Experimental results indicate that by using $Z_0/Z_{0\infty}$ values (rather than using either Z_0 or $Z_{0\infty}$) as a basis for the

[11] S. B. Cohn, "Properties of ridge waveguide," Proc. IRE, vol. 35, pp. 783–788; August, 1947.

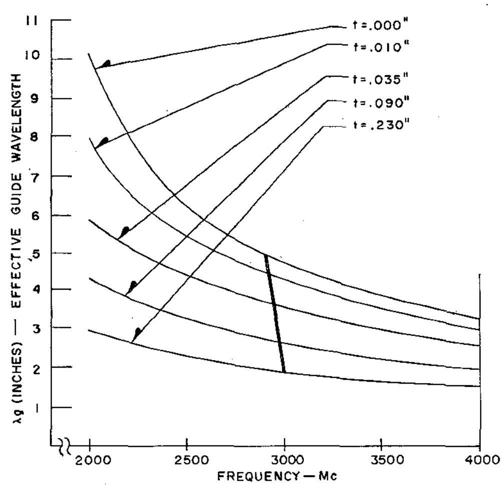


Fig. 6—Effective guide wavelength vs frequency for DR-37 double-ridge waveguide, with $\epsilon_1=1.00$ and $\epsilon_2=9.60$.

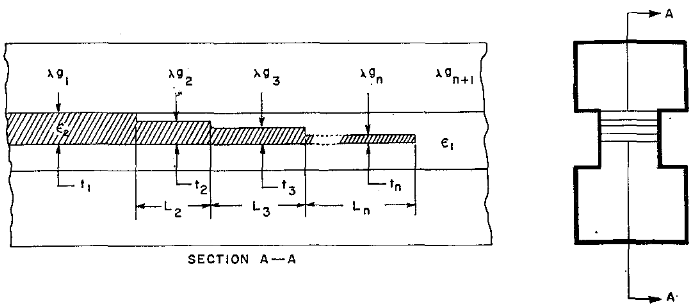


Fig. 7—Partially filled double-ridge waveguide step transition.

design of a step transition, a better over-all response is obtained. It is convenient to define, through the use of (14), the following ratio:

$$C^S = \frac{\lambda_{g_{n+1}}}{\lambda_{g_1}} \tag{15}$$

where

C = a dimensionless constant,

S = the sum of a certain set of a_m constants as outlined by Cohn.[6] [See Appendix, Sections A and B-2), below],

$$\lambda_{g_{n+1}} = \frac{2\lambda_{gl}\lambda_{gh}}{\lambda_{gl}+\lambda_{gh}},$$

where

λ_{gl} = longest guide wavelength at $t=0$,

λ_{gh} = shortest guide wavelength at $t=0$,

$$\lambda_{g_1} = \frac{2\lambda_{g1h}\lambda_{g1l}}{\lambda_{g1h}+\lambda_{g1l}},$$

where

λ_{g1l} = guide wavelength for the maximum dielectric thickness at the lowest frequency,

λ_{g1h} = guide wavelength for the maximum dielectric thickness at the highest frequency.

As an example in Fig. 6, $\lambda_{gl}=10.12$ inches, $\lambda_{gh}=3.24$ inches, $\lambda_{g1l}=2.93$ inches, and $\lambda_{g1h}=1.43$ inches.

Upon the solving of (15) for C, the intermediate effective guide wavelengths are computed as

$$\begin{aligned}\lambda_{g2} &= \lambda_{g1}C^{a_1}\\ \lambda_{g3} &= \lambda_{g1}C^{a_1+a_2}\\ &\vdots\\ \lambda_{gn} &= \lambda_{g1}C^{a_1+a_2+\cdots+a_{n-1}}.\end{aligned} \tag{16}$$

In Fig. 7, the dielectric thickness, t_n, for each transformer is obtained by use of curves such as those shown in Fig. 6, where $\lambda g_{n+1}=4.92$ inches and $\lambda_{g1}=1.92$ inches. When these two points are connected by a straight line, the intersection of this line with the appropriate value of λ_{gn}, from (16), yields the thickness to be used for the nth transformer. For example, if λ_{gn} is computed from (16) to be 3.25 inches, from Fig. 6, the transformer thickness, t_n, is seen to be 0.050 inch.

b) *Transformer length:* Each transformer length for the partially filled double-ridge waveguide is given by

$$L_n = \frac{\lambda_{gn}}{4} \tag{17}$$

where λ_{gn} is as defined in (16).

c) *Theoretical VSWR:* The theoretical VSWR for the stepped transformer has been discussed above [in Section II-A, 1c)]. For the double-ridge waveguide step transition, the theoretical VSWR is obtained by simply replacing

$$\frac{Z_{n+1}}{Z_1} \quad \text{by} \quad \frac{\lambda_{gn+1}}{\lambda_{g1}}$$

in (7).

III. Experimental Results

A. Step Transition for Partially Filled Coaxial Line

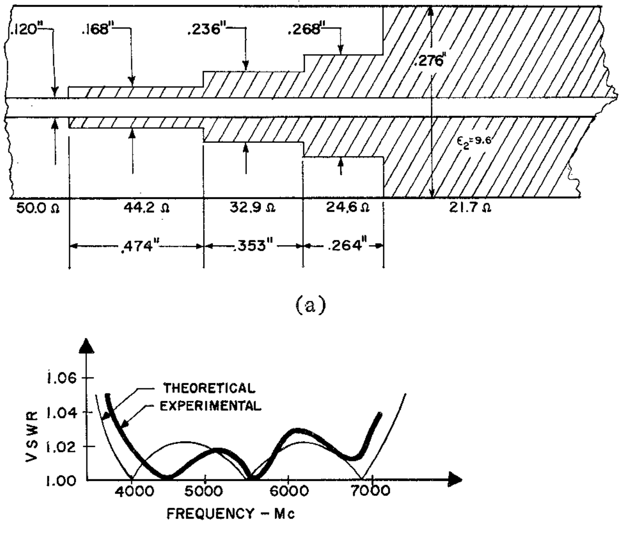
Data have been obtained for a four-step coaxial transition, shown in Fig. 8(a), with a design range of 4000 to 7000 mc. A 21.7-ohm end impedance was matched to a standard air-filled, $\frac{3}{8}$-inch, —50-ohm coaxial line. In this design $\theta=180°$ and $\epsilon_2=9.6$.

The maximum theoretical VSWR is 1.023, while the maximum measured VSWR in the 4000- to 7000-mc design region was 1.033. The band response is shown in Fig. 8(b).

B. Step Transition For Partially Filled Double-Ridge Waveguide

A five-step transition, shown in Fig. 9(a), has been tested for DR-37 double-ridge waveguide. The frequency range covered was 2000 to 4000 mc, which would result in a value of

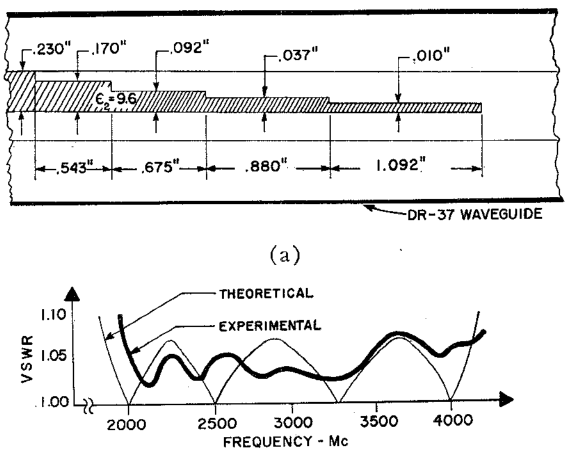
$$p = \frac{\lambda_{gl}}{\lambda_{gh}} = \frac{10.12}{3.24} = 3.12 \qquad \text{(see Fig. 6)}.$$



(a)

(b)

Fig. 8—(a) Design for a partially filled coaxial line, and (b) experimental data.



(a)

(b)

Fig. 9—(a) Design for a partially filled double-ridge waveguide, and (b) experimental data.

However, as an added safety factor, a value of $p = 3.30$ was chosen. The maximum dielectric thickness used was 0.230 inch, with a dielectric constant of 9.6.

The step transition was placed on the input side of a waveguide isolator. The ferromagnetic material did not substantially change the effective characteristics of the partially filled waveguide. For the five-step double-ridge waveguide transition, the maximum theoretical VSWR is 1.081. As can be seen from the band response in Fig. 9(b), the measured VSWR (isolator input VSWR) is less than 1.090 throughout most of the 2000- to 4000-mc band. For a design such as that shown in Fig. 9(b), it should be noted that the thickness of any particularly thin transformer may be increased by simply using a lower dielectric constant material while maintaining the same transformer length.

IV. Conclusion

A theoretical analysis has been made of the effective guide wavelength and characteristic impedance of partially filled transmission lines. Although proximity effects have been neglected in this analysis, experimental results are found to closely approximate the theory.

By use of the equations presented, step transitions may be designed for partially filled (dielectric loaded) transmission lines.

Appendix

A. Determination of S

In (4) and (15), where $S = \sum a_m = a_1 + a_2 + a_3 + \cdots + a_n$, the value of S is determined by 1) the bandwidth p and 2) n, the number of steps used the in step-transition design. A method of computing the a_m values is described by Cohn.[6] For a five-step transition ($n = 5$) and a bandwidth p equal to 3.30, the value of S would be

$$S = a_1 + a_2 + a_3 + a_4 + a_5.$$

$$S = 1 + 1.787 + 2.194 + 1.787 + 1.000 = 7.768.$$

B. Bandwidth

1) *Partially Filled Coaxial Line:* In the design of partially filled coaxial line step transitions, the bandwidth p is given by the following equation:

$$p = \frac{\lambda_l}{\lambda_h} \tag{18}$$

where

λ_l = longest free-space wavelength,
λ_h = shortest free-space wavelength.

2) *Partially Filled Double-Ridge Waveguide:* In the design of partially filled double-ridge waveguide step transitions, the bandwidth p is given by

$$p \approx \frac{\lambda_{gl}}{\lambda_{gh}} \tag{19}$$

where

λ_{gl} = guide wavelength at the lowest frequency in air-filled double-ridge waveguide,
λ_{gh} = guide wavelength at the highest frequency in air-filled double-ridge waveguide.

The computed value for p in double-ridge waveguide will usually be slightly higher than the precise value required. However, a somewhat still higher value of p is frequently used to provide a small safety factor in the design.